B.Sc. Part-I

Paper-I

Theory of Relativity

Dr. Shiva Kant Mishra
Dept of Physics HDJC

— Length Contraction —

We made mention earlier that as a result of special relativity absolute space and absolute time do not exist. Rather, the two are interdependent and any change in time must cause a change in space (length or distance) and vice versa. How can this be explained?

The fact that speed is the ratio of length to time means that both length and time must change, in just the right proportion, in order to ensure the speed of light remain constant.

We therefore, can determine the change in length by using the result we found for time dilation.

Let us suppose Earth-bound observers watch a spaceship that travels with velocity v between two distant points. The distance between the two points as measured by an earth-bound observer is L_p, and the time for the trip is t.

We can then state

$$L_p = vt \quad \text{——— (1)}$$

However $$t = \frac{t_p}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{——— (2)}$$

Substituting t in eqn (2) into eqn (1) gives us

$$L_p = v \frac{t_p}{\sqrt{1 - \frac{v^2}{c^2}}} \rightarrow vt_p = L_p \sqrt{1 - \frac{v^2}{c^2}} \rightarrow$$

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but vt_p is the distance L measured in the spaceship, so $L = L_p\sqrt{1-v^2/c^2}$

The distance the observers in the space ship measure compared to the distance that observers on Earth measure is therefore shorter, since $\sqrt{1-\frac{v^2}{c^2}} < 1$.

The shortening of distance is called Length Contraction and it is not limited to the "space" between two points. The same contraction occurs for material objects. For example - if one inertial frame moving with a velocity v, passed another inertial frame containing a meter stick, the observers in these frames will not agree on the length of the meter stick. Those at rest relative to the meter stick will claim it does measure one meter. The occupants of the moving frame, on the other hand, will claim it is shorter than one meter.

It must be noted that length contraction is observed only in the direction of motion of the moving object. in our example with a meter stick, the width of the meter stick does not change, only its length.

Teacher's Signature

Q.1. An earth bound observer measures the distance to the Star Sirius as 8.7 light-years (Ly) (a light year is the distance that light travels in one year). What distance between the earth and Sirius would be measured by an observer on a space ship moving at speed 0.8c ?

Ans. Using the result derived above, we have

$$L = L_p\sqrt{1-\frac{v^2}{c^2}} = 8.7\sqrt{1-\frac{(0.8)^2c^2}{c^2}}$$

$$= \underline{5}$$

Q.2 If photons were capable of perceiving the effect of length contraction, what length would they assign to an object which measures 1 meter in its proper reference frame?

Ans we can see that the time and length in special relativity are changed by a factor of $\sqrt{1-v^2/c^2}$. In the case of length, we get the rather amazing result that from the point of view of light, all material objects contract to zero length since $v = c$. Therefore, the meter stick, as well as any object, has zero length as measured by our perceptive photons. From photon's point of view, then it takes zero time to get anywhere. You will never convince a photon that they are not on time.